Spiral Science & Math Manipulative

Teacher's Guide

Science-Based Math Activities Exploring

Fibonacci Series • Naturally Occuring Spirals
Ratios • Proportionality • Number Patterns
Graphing • Golden Mean
Units of Measure

Student Worksheets & Other Resources available at:
www.Spiral-Science.com

Innovative Educational Resources LLC

Creating enduring educational experiences sparked by student discovery

Teacher's Guide

Lessons and Activities

TABLE OF CONTENTS		
Preface		2
Overview		3
Guide for Classroom Use		3
Relationship to Curriculum Standards		3
Background Materials		4
Online Resources		5
Lesson 1 – Introduction		6
Lesson 2 – Ratios & Proportionality		7
Lesson 3 – Graphing Data & Trending		8
Lesson 4 – Rationalizing Fractions		9
Lesson 5 – Measurement Instruments		10
Lesson 6 – Data Analysis & Probability		11
Other Inquiry Based Exercises		12
Companion Products, Credits, & References		13

Innovative Educational Resources, LLC
Stewartsville, New Jersey 08886
(908) 521-0588
info@spiral-science.com

www.spiral-science.com

© Copyright 2009 by Innovative Educational Resources, LLC

Preface

The Spiral Science & Math Manipulative along with this guide has been developed to provide many useful science based mathematic activities and lessons that can be implemented easily with little preparation.

The teacher and poster size Spiral Science and Math Manipulative are intended to be used by teachers to guide students through a series of science based mathematic activities.

These exercises can best be performed with students working in small groups using the student size manipulative (purchased separately or in kits). Teachers can also consider having students construct a replica using construction paper or card stock as part of a related arts project.

A few ideas for optional activities include:

1. Have students draw examples of spirals on one side of each square. Challenge their creativity and curiosity in nature by asking them to draw from examples in nature.
2. Ask students to cut out 2 or 3 additional squares larger than 13 x 13 cm that fall into the number pattern.

The lessons and activities included in this guide are geared to older elementary and middle school curriculum content, however, the Spiral Science & Math Manipulative can be adapted for use with younger students.

Uses in early elementary classrooms include:

- ☑ Counting Sequences
- ☑ Simple Addition & Subtraction Exercises
- ☑ Introduction to Square, Rectangle, & Spiral Forms
- ☑ Basic Measurement Exercises

The Spiral Science & Math Manipulative also serves as a pathway to many other educational lessons and classroom activities that can be developed from the body of knowledge that exists with the Golden Mean, Fibonacci Series, and naturally occurring spiral patterns.

Please visit our website at **www.spiral-science.com** to learn more about the important educational applications of this product or to download free activity sheets and lesson plans.

OVERVIEW

The Golden Mean, Fibonacci Series, and Spiral Patterns emerge to offer students a discovery based opportunity to engage in a series of fun educational activities that explore the interrelationship that exists between science and mathematics.

Scientific and mathematical concepts converge within the Spiral Science & Math Manipulative to provide the educational basis for improving student performance in areas related to ratios, proportional reasoning, rationalizing fractions, units of measure, graphic representation of data, and the selection of measuring instruments. The inquiry driven lessons provide many creative learning opportunities to draw student attention toward multiple fields of scientific study, including; Astronomy, Botany, Biology, Mathematics, Meteorology, and Paleontology.

GUIDE FOR CLASSROOM USE

The Spiral Science & Math Manipulative is designed for use across multiple grade levels and has been found to effectively motivate student participation in hands-on math based exercises. The earliest implementation can occur once students have begun to learn addition and subtraction when the magnetic tiles can be used to set up simple math problems. It can also lead to counting exercises that stimulate interest in various areas of science within this same population of children.

The Spiral Science & Math Manipulative provides a fun and exciting way to engage students in the application of math concepts, especially during grades 5th through 8th when they begin to encounter increasing difficulty with math concepts at a time when their science activities also rely upon these important math skills.

RELATIONSHIP TO CURRICULUM STANDARDS

The design of the Spiral Science & Math Manipulative is consistent with multiple elements of the National Science Education Standard and the Principles and Standards for School Mathematics (NCTM) to respond to learning difficulties documented by The American Association for the Advancement of Science and other organizations.

Most importantly, the Spiral Science and Math Manipulative serves to give teachers an opportunity to improve the student's ability to conduct scientific inquiry while increasing their science-based math skills during a fun and insightful journey through the realm of natural science.

The Spiral Science & Math Manipulative focuses student attention on the value gained through scientific inquiry, as well as, the use of appropriate tools and techniques to gather, analyze, and interpret data. It also establishes a coordinated link between a teacher's science and math programs.

This strategy is aligned to the Science Education Program Standards that indicate science programs should be "connected to other school subjects" and be "coordinated with the mathematics program to enhance student use and understanding of mathematics in the study of science."

Additionally, the Spiral Science & Math Manipulative engages eight (8) of the twelve (12) items (Grades K-8) listed in Table 7.1 of the National Science Education Standard as examples of mathematics students should use and understand, including the ability to:

- Measure, collect, and organize data.
- Recognize and describe patterns.
- Develop skills of estimation and judgment.
- Represent situations verbally, numerically, graphically, geometrically, or symbolically.
- Use estimations.
- Develop and use tables, graphs, and rules to describe situations.
- Use statistics to describe, analyze, evaluate, and make decisions.
- Create experimental and theoretical models of situations involving probabilities.

BACKGROUND MATERIAL

The Spiral Science & Math Manipulative is patterned after a tiling of squares equal in length and width to the numbers contained in the Fibonacci Series. Starting with 2 squares of equal size, the manipulative forms a rectangle with a ratio that approaches the Golden Mean in just the first several numbers of the series.

This well-known number sequence, first originating in India, was applied by Fibonacci in a problem designed to calculate generational increases in a population of rabbits given certain rules and has been adapted in puzzles and problems dealing with cows, honeybees, and the sharing of secrets. The progression of the series can continue uninterrupted to provide the framework leading to the construction of a logarithmic spiral.

The Golden Mean is a central and recurring theme in mathematics that has been shown to have a strong influence in both Art and Architecture with an aesthetic basis that seems to echo throughout nature. The Golden Mean, or in Greek the term *Phi* (**Φ),** is the number 1.6180339… and is frequently applied as a ratio used to divide a line or establish the proportion of sides of a "Golden Rectangle."

The inverse of the Golden Mean; **1/Φ** is equal to 0.6180339…. This is unique when we stop to recognize that the inverse of other numbers, such as 1.333333…. with an inverse of 0.75, generally bear no visual similarity to their original number. The use of both terms, **Φ and 1/Φ**, enables us to guide students between the 'whole' and the 'part' quite easily.

The Golden Mean (Φ) can be derived algebraically through the equations:

$\Phi = (\sqrt{5}+1)/2$ or

$1/\Phi = (\sqrt{5}-1)/2$

More important, the Golden Mean Line and Golden Rectangle can also be constructed using basic geometric objects and terminology consistent with the academic level of students using the Spiral Science and Math Manipulative.

ONLINE LEARNING RESOURCES

Innovative Educational Resources, LLC is committed to extending the value of your purchase by offering free online access to additional learning resources, making the Spiral Science & Math Manipulative easy to use and simple to understand.

Available online resources include:

- ☑ Downloadable Data Sheets & Graphing Templates.
- ☑ Digital Images of other Naturally Occurring Spirals.
- ☑ Student Activity Sheets.
- ☑ Instructions for Related Experiments.

Lesson 1 – Introduction in Three Parts

Objective

Part 1: Introduction to Manipulative

- ☑ Students will be able to decide which numbers are missing from the sequence.
- ☑ Students will be able to predict which numbers are next in the sequence.

Part 2: The Golden Spiral

- ☑ Students will understand the definition of a Golden Spiral.
- ☑ Students will be able to use another name for Golden Spiral.

Part 3: Naturally Occurring Spirals

- ☑ Students will be able to list other examples of naturally occurring spirals.

Learning Activity

Part 1: Introduction to Manipulative

- ☑ Introduce the Spiral Science & Math Manipulative by placing the separated pieces number side up in sequential order, leaving out one or more numbers.

  1, 1, 2, 3, __, 8, 13, __, __, __

- ☑ Have students look at the number sequence. Can they tell which number is missing? When the missing number is given, challenge the student to predict the next numbers in the sequence.
- ☑ When students understand the number sequence, assemble the manipulative to reveal the pattern.

Part 2: The Golden Spiral

- ☑ Teachers should explain that placing the manipulative together forms a golden spiral. Give the definition of a Golden Spiral and other names that can be used for Golden Spiral.

Part 3: Naturally Occurring Spirals

- ☑ The "spiral side" of the Spiral Science & Math Manipulative gives different examples of spirals found in nature. Flip one of the sections over to reveal a spiral found in nature. Challenge students to come up with other examples of spirals found in nature.
- ☑ List their ideas on the board, and flip over the appropriate piece when a student says one of the spirals illustrated on the manipulative.

Lesson Plan Expansion

- ❖ Have students consider other geometric forms found in nature.
- ❖ Explore tessellations and discuss fractals.
- ❖ Ask students to write a paragraph or more about the science associated with one of the naturally occurring spirals illustrated on the manipulative.

Vocabulary

Spiral
Archimedes Spiral
Golden Spiral
Logarithmic Spiral
Growth Spiral
Tessellations
Square
Ratio
Proportion
Radius
Fractals
Symmetry

Materials & Resources

Spiral Science & Math Manipulative
Student Activity Sheets (Downloadable)
Vocabulary Definitions

Lesson 2 – Ratios & Proportionality

Objective

Exploring the World of Ratios

- ☑ Students will be able to conceptualize fractions and decimals.
- ☑ Students will understand the relationship between 'parts' and 'wholes'.
- ☑ Students will find the proportion of a series of related geometric objects.
- ☑ Students will learn to use the metric system.

Learning Activity

Determining the ratio and proportion of various physical objects and fluids are common math-based activities that frequently occur during science lessons. This activity provides a tangible "hands-on" learning opportunity that enables students to become more proficient in the use of measuring instruments and overcome their learning difficulties relating fractions to decimal numbers.

- ☑ Using the metric system, instruct students to measure the length and width of the manipulative each time they add a new square to build up the rectangle.
- ☑ Students create a data table then record their measurements and the ratio they calculate by dividing the length by the width.
- ☑ Students should write the ratio to at least the third decimal place.

Interval	Length	Width	Ratio L/W
1	1	1	
2	2	1	
3			

Teacher's Note -

Downloadable Datasheets are available at www.spiral-science.com.

Keep the completed data table for lesson 3.

Do not check student work for accuracy until after lesson 3.

Lesson Plan Expansion

- ❖ Have students measure and compare the length of each of the bones in the same finger from smallest to largest. Try having them multiply the smallest bone by 1.618 and see how closely the result matches the measured result for the next biggest bone. X-Ray Images are available at www.spiral-science.com.

Vocabulary

Golden Mean
Ratio
Proportion
Metric System

Materials & Resources

Spiral Science & Math Manipulative
Vocabulary Definitions
Metric Ruler
Blank Data Table (Downloadable)

Lesson 3 – Graphing Data & Looking for Trends

Objective

Graphing and Evaluating Data

- ☑ Students will understand how to construct a graph.
- ☑ Students will understand that the points on a graph are related symbols.
- ☑ Students will evaluate data on a graph, see patterns, & compare results.

Learning Activity

- ☑ Students will create a graph with an x and a y axis.
- ☑ The calculated ratio from lesson 2 will be plotted on the y axis.
- ☑ The interval will be on the x axis.
- ☑ Have students plot the data points without connecting them and record any trends that are spotted.
- ☑ After recording the trends have students connect the data points using a ruler.
- ☑ When looking at the findings students should consider:
  - What shape has the graph taken?
  - If any new trends appear?
- ☑ Using different colored pencils have students connect the minimum and the maximum points.
- ☑ Ask students if they can find any math errors using the graph?

Teacher's Note -

Downloadable graph paper is available at www.spiral-science.com.

The general shape of the resulting graph will be a saw-tooth pattern with decreasing separation between the points.

Errors in lesson 2 should become evident during graphing in lesson 3.

Incorrect data will show as:

- 2 or more consecutive maximum or minimum points.
- 1 or more data points that are significantly less than or greater than the following points on the graph.

Lesson Plan Expansion

- ❖ Discuss how developing the skill to identify trends and expected results is not only important in scientific investigations but can also be useful in completing homework assignments and reviewing test results.

Vocabulary

Experimental Data
Interval Scale
Y axis
X axis

Materials & Resources

Spiral Science & Math Manipulative
Results from Lesson 2
Vocabulary Definitions
Graph Paper (Downloadable)
Ruler
Colored pencils

Lesson 4 – Working with Fractions

Objective

Comparing Different Measuring Instruments

- ☑ Students will be able to use and understand the customary English Scale Ruler.
- ☑ Students will be able to conceptualize fractions and decimals.
- ☑ Students will understand ratios and proportions.

Learning Activity

The concepts of ratio and proportionality initially developed in Lesson 2 are revisited in a revised version using an customary English Scale Ruler. This subtle difference leads to 2 additional math computations that are required to rationalize the fraction before the proportion can be calculated.

- ☑ Teachers instruct students to measure the length and width of the manipulative using an English Scale Ruler each time they add a new square to build up the rectangle.
- ☑ Students create a data table then record their measurements and the ratio they calculate by dividing the length by the width.
- ☑ Students should write the ratio to at least the third decimal place.

Teacher's Note -

This activity is separate from the previous lesson and should be conducted without calling attention to the data from lesson 2.

Keep the table for lesson 5 and do not check student work for accuracy until after lesson 5.

Lesson Plan Expansion

- ❖ Have students measure the length of various physical objects and the volume of water using both the Metric and Customary English units of measure.

- ☑ Create an exercise that requires them to add, subtract, multiply & divide the quantities.
- ☑ Consider using experiments that explore the density of various liquids or demonstrate other basic physical properties of matter.

Vocabulary

English Scale
Fraction
Decimal

Materials & Resources

Spiral Science & Math Manipulative
Customary English Scale Ruler
Vocabulary Definitions
Blank Data Table (Downloadable)

Lesson 5 – Selection of Measuring Instruments

Objective

To establish a line of inquiry that asks students to engage and explore their learning experiences relative to the selection of measuring instruments.

☑ · Students will understand how to follow a scientific method relative to:

- Record Keeping.
- Communication.
- Selection of Measurement Tools.
- Control and Conditions.

Learning Activity

☑ Students will have a class discussion on the variables between lessons 2 and 4 in relation to record keeping, communication, conditions, and the selection of measuring instrument.

☑ Students will discuss how each of these variables impact their findings found in lessons 2 and 4.

☑ Have students develop a hypothesis and investigation to confirm or disprove whether lesson 2 or 4 was more accurate.

☑ Introduce the importance of the scientific method in: record keeping, communication, selection of measurement tools, control and conditions.

☑ Reinforce the following concepts:

- When science investigations are exactly as they were done before, results are expected to remain unchanged.
- When the results of scientific investigations vary by large amounts it is necessary to figure out why.
- Similar investigations provide different results because of unexpected differences, the method used, circumstances, environment, measurement error, and uncertainty.
- When different results occur, the scientific challenge to determine the significance of the variation often leads to additional investigations.

☑ Be certain to explore the following concepts:

- Are lessons 2 and 4 identical or similar experiments?
- Is it correct to assume that because the measured object remained unchanged that the results should remain unchanged?
- What were the specific differences between the two lessons?
- Did their work during Lesson 2 bias their results in Lesson 4?

Lesson Plan Expansion

❖ Ask the students or another class to conduct independent experiments to test the class hypotheses.

Vocabulary

Scientific Method
Variable
Control
Hypothesis

Materials & Resources

Lesson 2 Results
Lesson 4 Results

Lesson 6 – Data Analysis & Probability

Objective

To illustrate how numerical data can be used to describe and compare results of experimental activities based upon the mathematical concepts of mean, median, and mode.

- ☑ Students will learn how numerical data can be used to describe and compare results of experiments.
- ☑ Students will understand mathematical concepts: mean, median, and mode.
- ☑ Students will learn to graph a histogram.

Learning Activity

Student Data

- ☑ The teacher will work with students to collect data from lesson 2 into a single set of data with the "interval" as column heading and "students" as row heading.

	Interval						
Student	1	2	3	4	5	6	7
Maya							
John							
Brady							
Sue							
Maximum							
Minimum							
Spread							

- ☑ Students will then:
  - Determine the minimum and maximum value from the data in each column.
  - Subtract the minimum value from the maximum value in each column to determine the spread (variability - range of the data).
  - This provides an indication of variability with respect to each interval.
  - Average the SPREAD to determine the average variability of the intervals and create a histogram with the Y-axis being quantity and the X-axis being ranges in value.
  - Repeat using the data taken in lesson 4.
  - Compare the average variability and histograms resulting from both sets of data to determine if it supports the hypotheses and conclusion made during lesson 5.

Lesson Plan Expansion

- ❖ Have students calculate the mean, mode and median using data from the activity.

Vocabulary

Average
Mean
Median
Mode
Spread
Analysis
Probability
Histogram
Comparison

Materials & Resources

Blank Datasheet (Downloadable)
Blank Graph Paper (Downloadable)
Data from Lesson 2 and 4
Hypothesis from lesson 5

Other Fun and Interesting Lesson Ideas

The Golden Mean and Fibonacci Series provides a fun and interesting ways to inspire students to make a closer examination of natural objects and explore the interaction between math and science.

Other related inquiry driven activities include:

Pentagram

- ☑ The Golden Mean reveals itself in the various segments of the pentagram star. How many examples can your class find?

Human Anatomy

- ☑ The relative length of the finger bones are given as just one example of where the golden ratio can be found in human anatomy. Facial features and the bones in the lower arm are cited as others. Have students sketch their silhouette and measure the distance between the key features including brow line, eyes, nose, and mouth. Take the distance as the ratio to the total distance between the top of the head and chin. Do they find examples of the golden ratio?
- ☑ The shape of the human Cochlea dictates the frequency response of our hearing. How might the shape and size of the Cochlea of other animals lead to differences in how they interpret sound?

Solar System

- ☑ The distance between the Sun and Mercury plus the distance from Mercury to Venus is said to equal the distance between Venus to Earth. Can your class confirm or disprove this? Examine Bode's Law and evaluate other objects within the Solar System against the Golden Mean?

Plant Growth

- ☑ Branching patterns, petal counts, and leaf positions offer an opportunity for students to make a closer examination of the structure of plant life.

Malthusian Principle of Population

- ☑ Published in 1798 by Thomas Malthus, the Malthusian Principle predicted that the human population would grow geometrically (1, 2, 4, 8, 16, etc) in comparison to the food supply that would only grow arithmetically (1, 2, 3, 4, 5, etc). This mathematic relationship was used to suggest that the human population would grow beyond its ability feed itself. In 2 centuries this would result in a 256 times increase in population in relation to a 9 times increase in food supply. Obviously a failed theory, one critic claimed "But science increases as fast as population."

  How do arithmetic, exponential, and geometric growth rates compare mathematically? In what other respects do scientists use these types of mathematical relationships to measure and track growth and decay?

Companion Products

Innovative Educational Resources, LLC

Individual Products -

- ☑ Spiral Science & Math Manipulative, Student Size
- ☑ Spiral Science & Math Manipulative, Teacher Size
- ☑ Spiral Science & Math Manipulative, Poster Size

Class Packs –

- ☑ Spiral Science & Math Manipulative, Starter
- ☑ Spiral Science & Math Manipulative, Standard
- ☑ Spiral Science & Math Manipulative, Deluxe

"Bundle of Fun" Inquiry Packs –

- ☑ Sound & Science "Bundle of Fun"
- ☑ Seeds & Science "Bundle of Fun"

Learn more about our products at: www.spiral-science.com.

Photo Credits

1 x 1 Squares	DNA, three dimensional example of an Archimedean Spiral *Photo Credit: Innovative Educational Resources LLC*
2 x 2 Square	Digitally enhanced drawings of the human cochlea. *Photo Credit: Innovative Educational Resources LLC*
3 x 3 Square	Ammonite *Photo Credit: Innovative Educational Resources LLC*
5 x 5 Square	Calla Lily and Ferns *Photo Credit: Innovative Educational Resources LLC*
8 x 8 Square	Twin Cyclones, Photograph taken via satellite November 2006 South of Iceland and West of Scotland. *Photo Credit: NASA*
13 x 13 Square	Spiral Galaxy Messier 81, Located about 12 million light years away. It is one of the brightest galaxies that can be seen from Earth through telescopes. *Photo Credit: NASA, ESA, the Hubble Heritage Team (STScI/AURA)-ESA/Hubble Collaboration, and W. Keel (University of Alabama, Tuscaloosa)*

References

American Association for the Advancement of Science. (2001), *Atlas of Science Literacy Volumes I & II* USA. A.A.A.S.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA.

National Research Council. (1996), *National Science Education Standards*. Washington DC. National Academy Press.

Spiral Science & Math Manipulative

The Spiral Science & Math Manipulative provides many useful science-based mathematic activities and lessons that can be implemented easily with little preparation to promote scientific inquiry among young students.

It is consistent with multiple elements of the *National Science Education Standard* and the *Principles and Standards for School Mathematics (NCTM) to improve student outcomes in science and math.*

It offers a fun, exciting way to engage students in the application of math concepts, especially during **grades 5th through 8th** when they begin to encounter increasing difficulty with math concepts at a time when their science activities also rely upon these critical important math skills, including:

- ☑ Use of Numerical Data to Compare Objects
- ☑ Identification of Numeric Sequences & Patterns
- ☑ Use of Numbers/Shapes to Describe & Predict Things
- ☑ Create and Use Tables, Graphs, & Rules
- ☑ Measure, Collect, & Organize Data
- ☑ Use Ratio & Proportion
- ☑ Develop Skills of Estimation & Judgment
- ☑ Analyze Data and Trends
- ☑ Spread Data on a Number Lines
- ☑ Rationalize and Translate Fractions
- ☑ Compare Similar Scientific Investigations
- ☑ Determine the Appropriate Measuring Instrument
- ☑ Use Mean, Median, & Mode to Describe a Data Set
- ☑ Use Statistical Methods to Analyze Data
- ☑ Improve Scientific Inquiry Skills

Uses in early elementary classrooms include:

- ☑ Counting Sequences
- ☑ Simple Addition & Subtraction Exercises
- ☑ Introduction to Square, Rectangle, & Spiral Forms
- ☑ Basic Measurement Exercises

Visit **www.spiral-science.com** to learn more about the important educational applications of this product or to download free activity sheets and lesson plans.